Calculating Simple Effects

A simple main effect (usually just called simple effect) is just the effect of one variable at levels of another variable. In Chapter 10 we had an example in which we'd measured the attractiveness of dates after no alcohol, 2 pints and 4 pints in both men and women. Therefore, we have two independent variables: alcohol (none, 2 pints, 4 pints) and gender (male and female). One simple effects analysis we could do would be to look at the effect of gender (i.e. compare male and female scores) at the three levels of alcohol. Let's look how we'd do this. We're partitioning the model sum of squares and we saw in Chapter 10 that we calculate model sums of squares using this equation:

$$SS_M = \sum n_k \left(\bar{x}_k - \bar{x}_{\text{grand}} \right)^2$$

For simple effects, we calculate the model sum of squares for the effect of gender at each level of alcohol. So, we'd begin with when there was no alcohol, and calculate the model sum of squares. So, the grand mean becomes the mean for when there was no alcohol, and the groups means are the means for men (when there was no alcohol) and women (when there was no alcohol). So, we group the data by the amount of alcohol drunk. Within each of these three groups, we calculate the overall mean, and also the mean of the male and female scores separately. These mean scores are all we really need. Pictorially, you can think of the data as in Figure 1.

We can then apply the same equation for the model sum of squares that we used for the overall model sum of squares, but we use the grand mean of the no alcohol data (63.75) and the means of males (66.875) and females (60.625) within this group:

No Alcohol		2 Pints		4 Pints	
Female	Male	Female	Male	Female	Male
65	50	70	45	55	30
70	55	65	60	65	30
60	80	60	85	70	30
60	65	70	65	55	55
60	70	65	70	55	35
55	75	60	70	60	20
60	75	60	80	50	45
55	65	50	60	50	40
60.625	66.875	62.50	66.875	57.500	35.625
Mean None = 63.75		**Mean 2 Pints = 64.6875**		**Mean 4 Pints = 46.5625**	

Figure 1: **Data arranged to look at the simple effect of gender at different levels of alcohol**

$$SS_{\text{Gender (No Alcohol)}} = \sum n_k \left(\overline{x}_k - \overline{x}_{\text{grand}}\right)^2$$
$$= 8(60.625 - 63.75)^2 + 8(66.875 - 63.75)^2$$
$$= 156.25$$

The degrees of freedom for this effect are calculated the same way as for any model sum of squares and that is they are one less than the number of conditions being compared ($k - 1$), which in this case when we're comparing only 2 conditions will be 1.

The next step is to do the same but for the 2 pints data. Now we use the grand mean of the 2 pint data (64.6875) and the means of males (66.875) and females (62.50) within this group. The equation, however, stays the same:

$$SS_{\text{Gender (2 Pints)}} = \sum n_k \left(\overline{x}_k - \overline{x}_{\text{grand}}\right)^2$$
$$= 8(62.50 - 64.6875)^2 + 8(66.875 - 64.6875)^2$$
$$= 76.56$$

The degrees of freedom are the same as in the previous simple effect, they are $k - 1$, which is 1 for these data. The next step is to do the same but for the 4 pints data. Now we use the grand mean of the 4 pint data (46.5625) and the means of females (57.500) and males (35.625) within this group. The equation, however, stays the same:

$$SS_{\text{Gender (4 Pints)}} = \sum n_k \left(\overline{x}_k - \overline{x}_{\text{grand}}\right)^2$$
$$= 8(57.50 - 46.5625)^2 + 8(35.625 - 46.5625)^2$$
$$= 1914.06$$

Again, the degrees of freedom are 1 (because we've compared two groups). As with any ANOVA, we need to convert these sums of squares to mean squares by dividing by the degrees of freedom. However, because all of these sums of squares have 1 degree of freedom, the mean squares will be the same as the sum of squares because we're dividing by 1. So, the final stage is to calculate an *F*-ratio for each simple effect. As ever, the *F*-ratio is just the mean squares for the model, divided by the residual mean squares. So, you might well ask, what do we use for the residual mean squares? When conducting simple effects we use the residual mean squares for the original ANOVA (the residual mean squares for the entire model). In doing so we are merely partitioning the model sums of squares and so keep control of the Type I error rate. For these data, the residual sum of squares was 83.036 (see section 10.2.6). Therefore, we get:

$$F_{\text{Gender (No Alcohol)}} = \frac{MS_{\text{Gender (No Alcohol)}}}{MS_R} = \frac{156.25}{83.036} = 1.88$$

$$F_{\text{Gender (2 Pints)}} = \frac{MS_{\text{Gender (2 Pints)}}}{MS_R} = \frac{76.56}{83.036} = 0.92$$

$$F_{\text{Gender (4 Pints)}} = \frac{MS_{\text{Gender (4 Pints)}}}{MS_R} = \frac{1914.06}{83.036} = 23.05$$

We can evaluate these *F* values in the usual way (they will have 1 and 42 degrees of freedom for these data). However, for the 2 pint data we can be sure there is not a significant effect of gender because the *F*-ratio is less than 1.